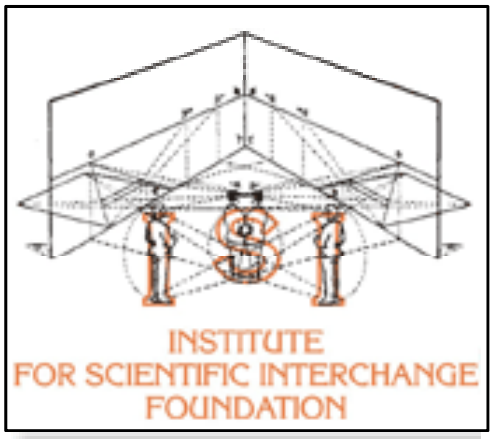
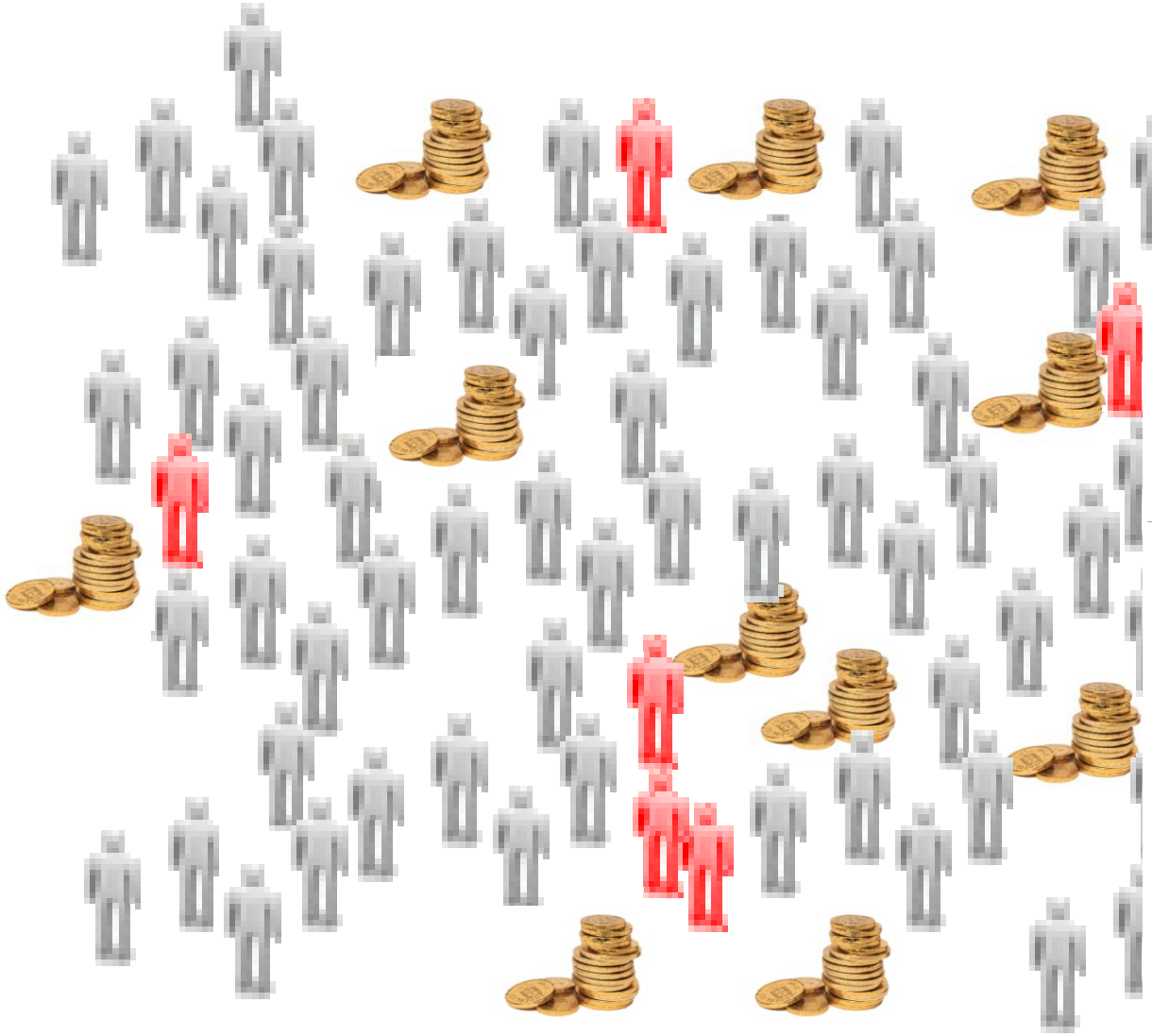
do the *rich* really take it all?

Vittoria Colizza
ISI Foundation, Turin, Italy

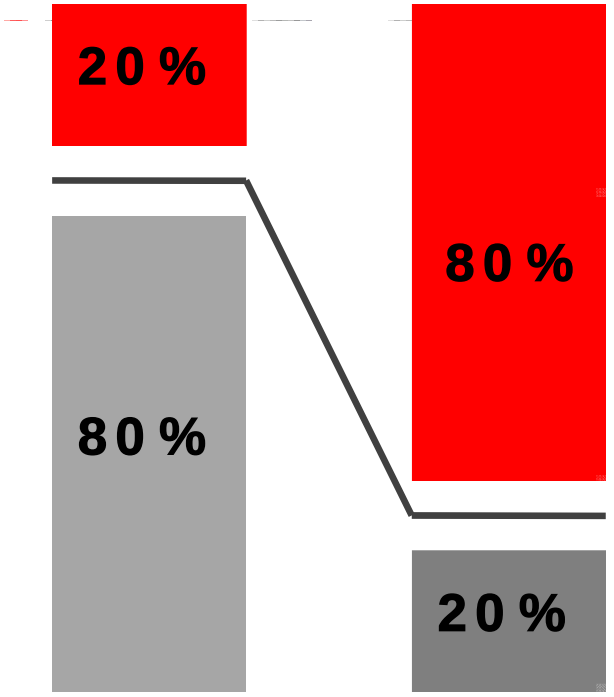


problem: prominence = control ?

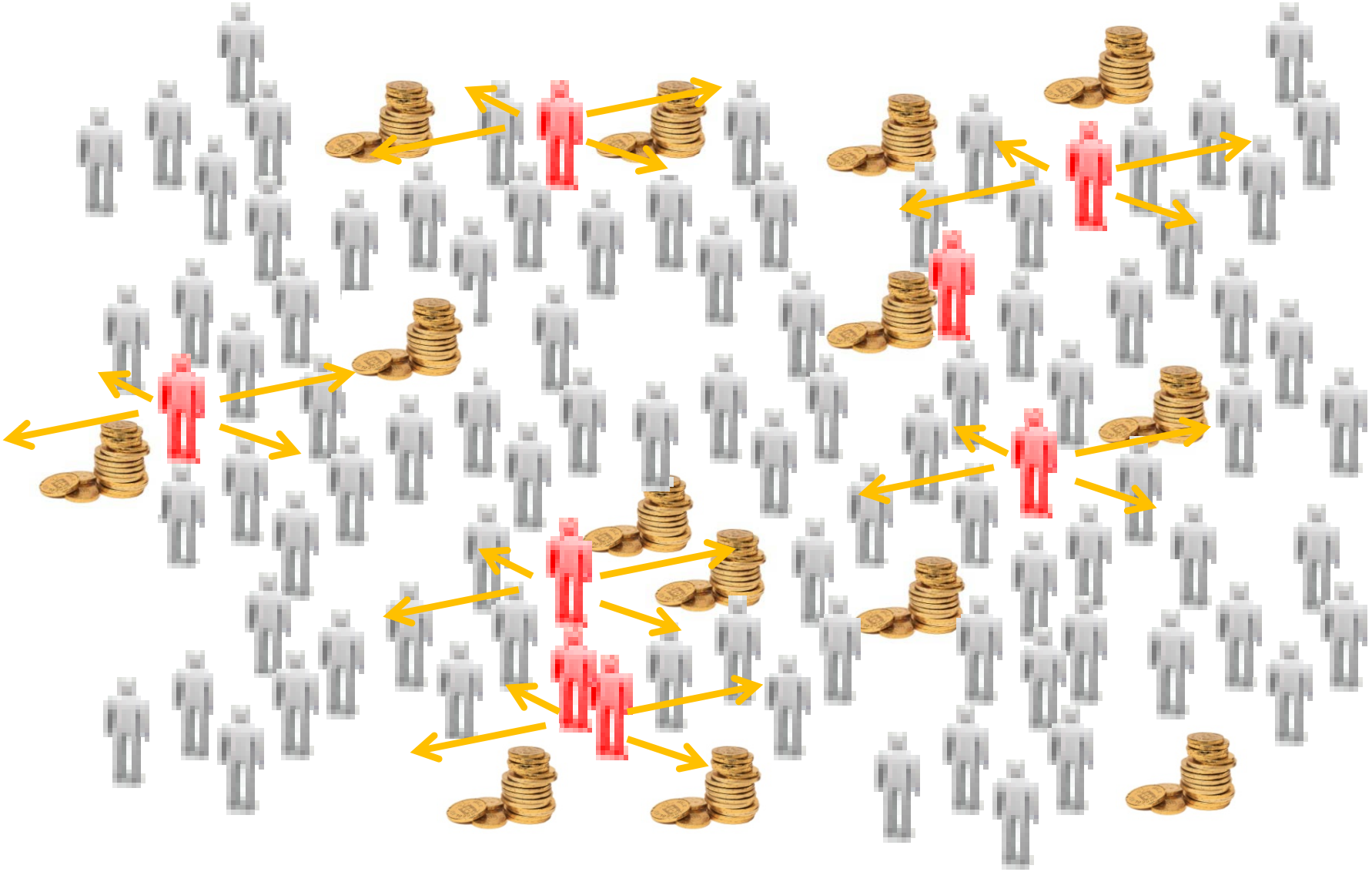


prominence

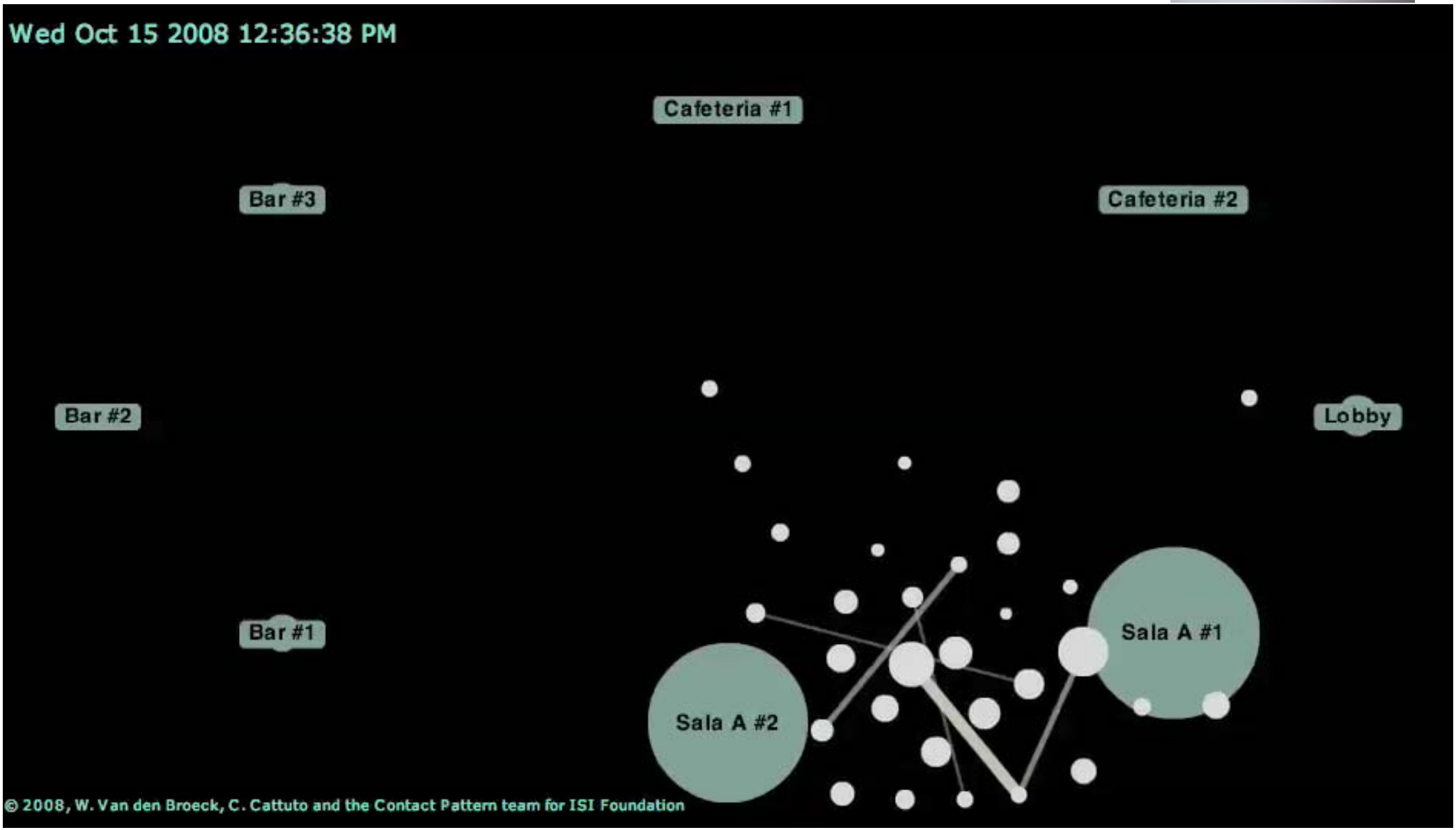
centralization of resources



problem: prominence = control ?



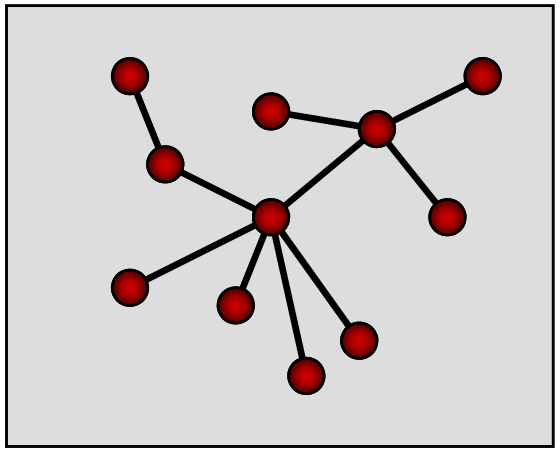
sociopatterns.org

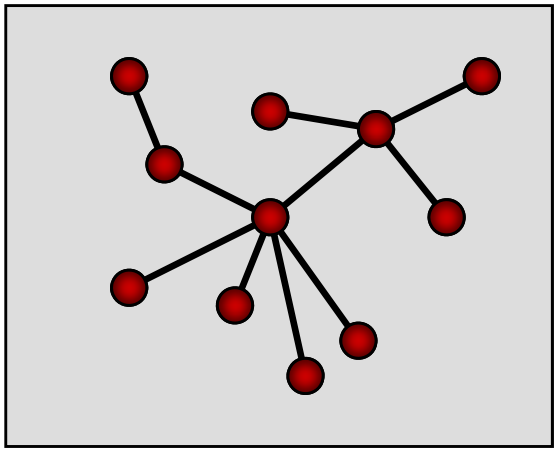


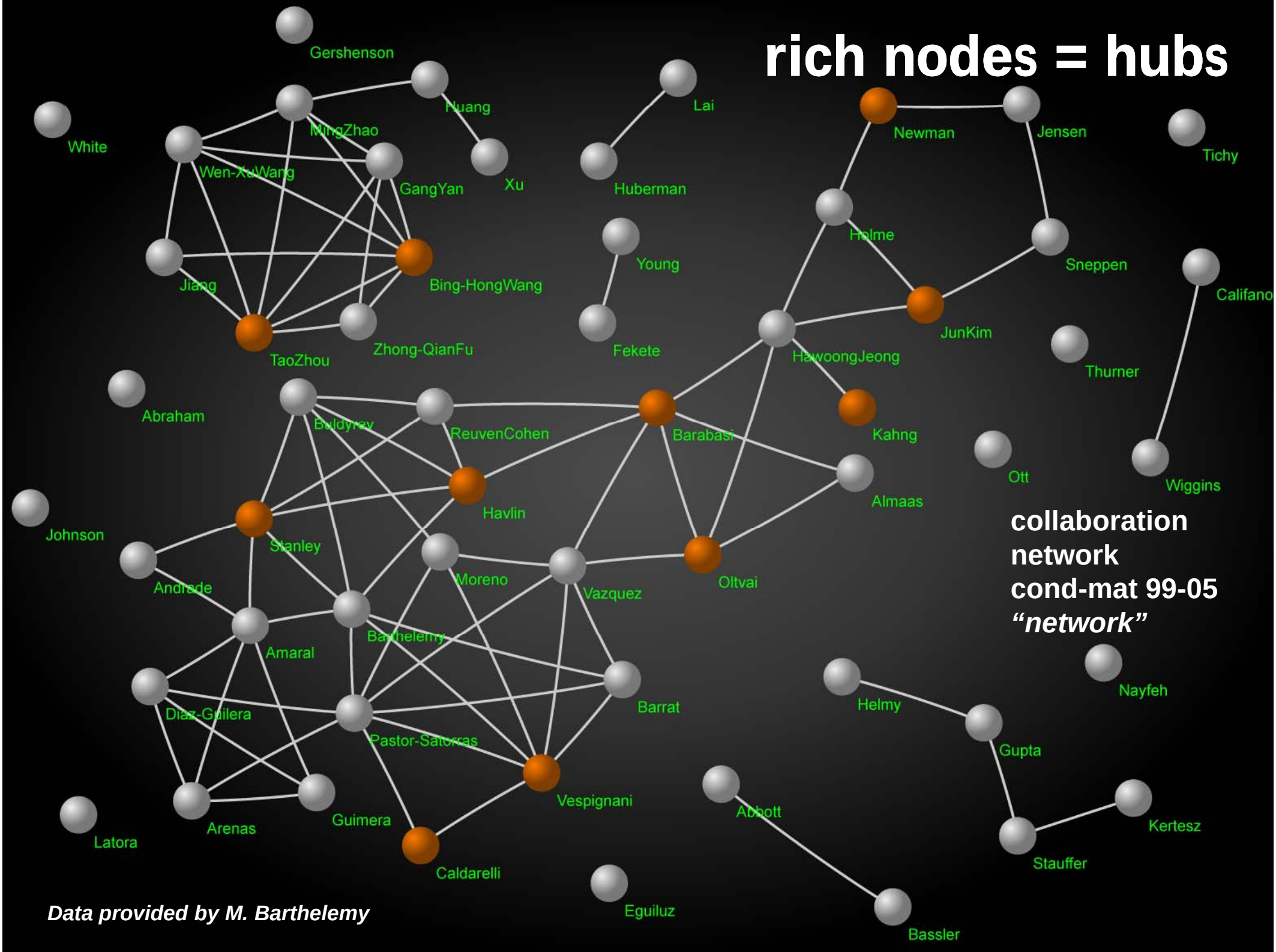
Barrat, Cattuto, Colizza, Pinton, Van den Broeck, Vespignani. arxiv:0811.4170

...what I'm not talking about

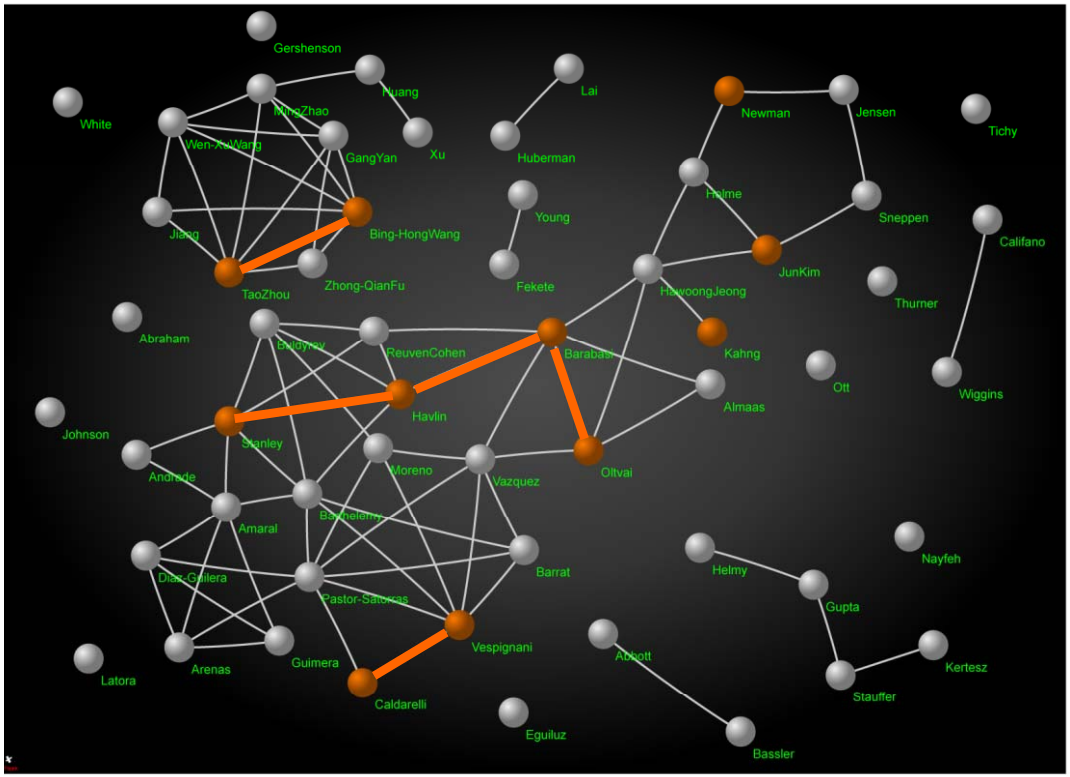
rich-club phenomenon $\neq$ 'rich-get-richer' mechanism






rich nodes = hubs
Gershenson
Huang
MingZhao
White
Wen-XuWang
GangYan
Xu
Lai
Huberman
Young
Jiang
Bing-HongWang
Fekete
TaoZhou
Zhong-QianFu
Newman
Jensen
Tichy
Holme
Sneppen
Califano
JunKim
HawoongJeong
Thurner
Abraham
Buldyrev
ReuvenCohen
Barabasi
Kahng
Ott
Wiggins
Havlin
Almaas
Johnson
Stanley
Andrade
Moreno
Vazquez
Oltvai
collaboration
network
cond-mat 99-05
"network"
Amaral
Barthelemy
Nayfeh
Helmy
Barrat
Diaz-Guilera
Pastor-Satorras
Gupta
Vespignani
Abbott
Kertesz
Latora
Arenas
Guimera
Stauffer
Caldarelli
Eguiluz
Bassler
Data provided by M. Barthelemy

rich-club coefficient



$$\phi(k) = \frac{2E_{>k}}{N_{>k}(N_{>k}-1)}$$

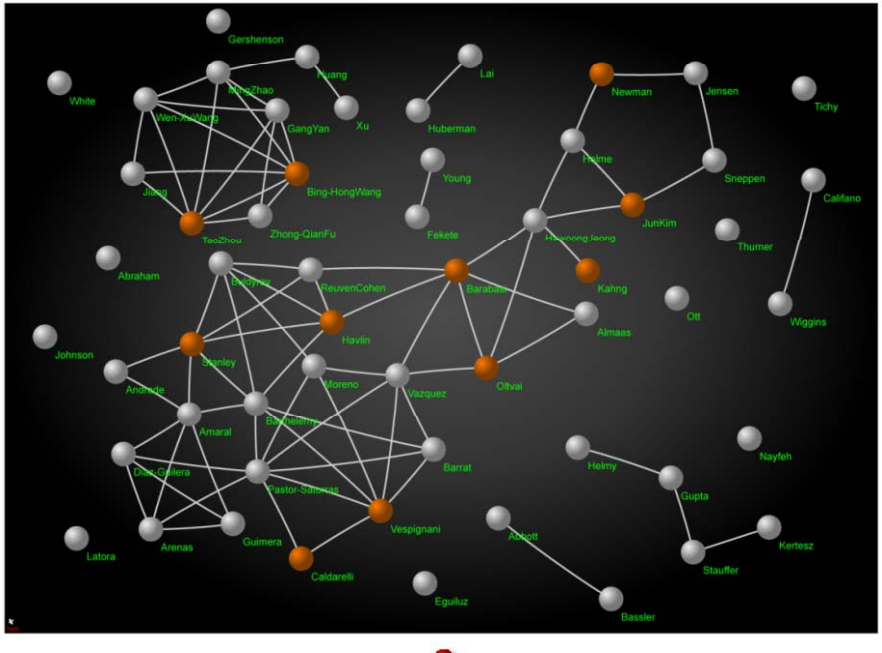
Zhou & Mondragon, IEEE Commun. Lett. (2004)

$N_{>k}$ = # nodes degree >k

$E_{>k}$ = # links among $N_{>k}$ nodes

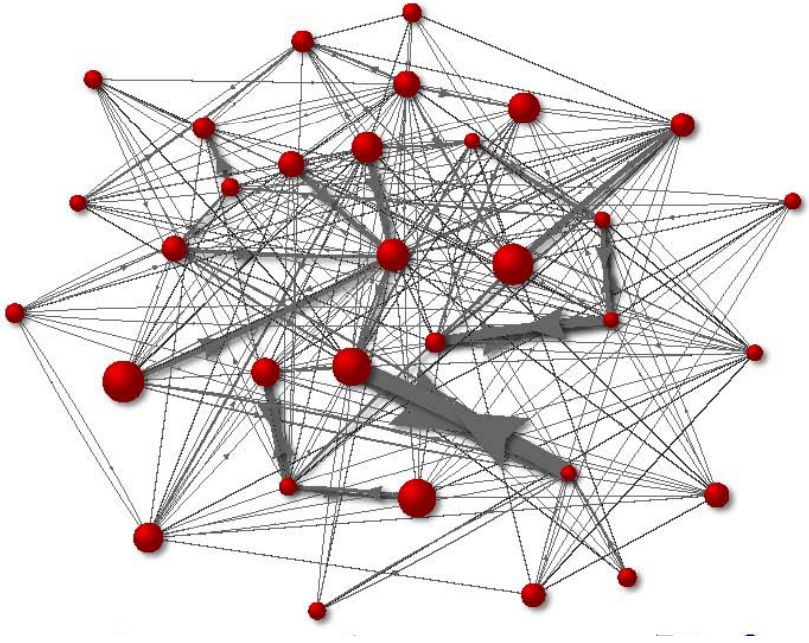
$$k = 22, \begin{cases} N_{>k} = 11 \\ E_{>k} = 5 \end{cases} \rightarrow \phi = \frac{2 \cdot 5}{11 \cdot 10} = 1/11$$

datasets: human communication / collab.



scientific collaboration network, cond-mat arxiv, 1995-1999

- 15,179 scientists
- 43,011 collaborations



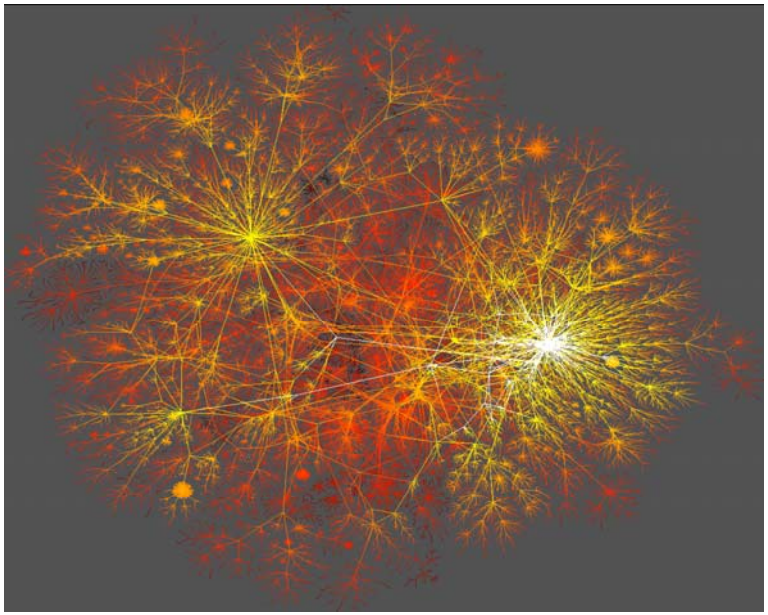
online social network, University of California, Irvine, April-October 2004

- 1899 college students
- 59,835 directed online messages

datasets: infrastructures

US airport network
(source: IATA)

- 676 airports
- 3523 routes

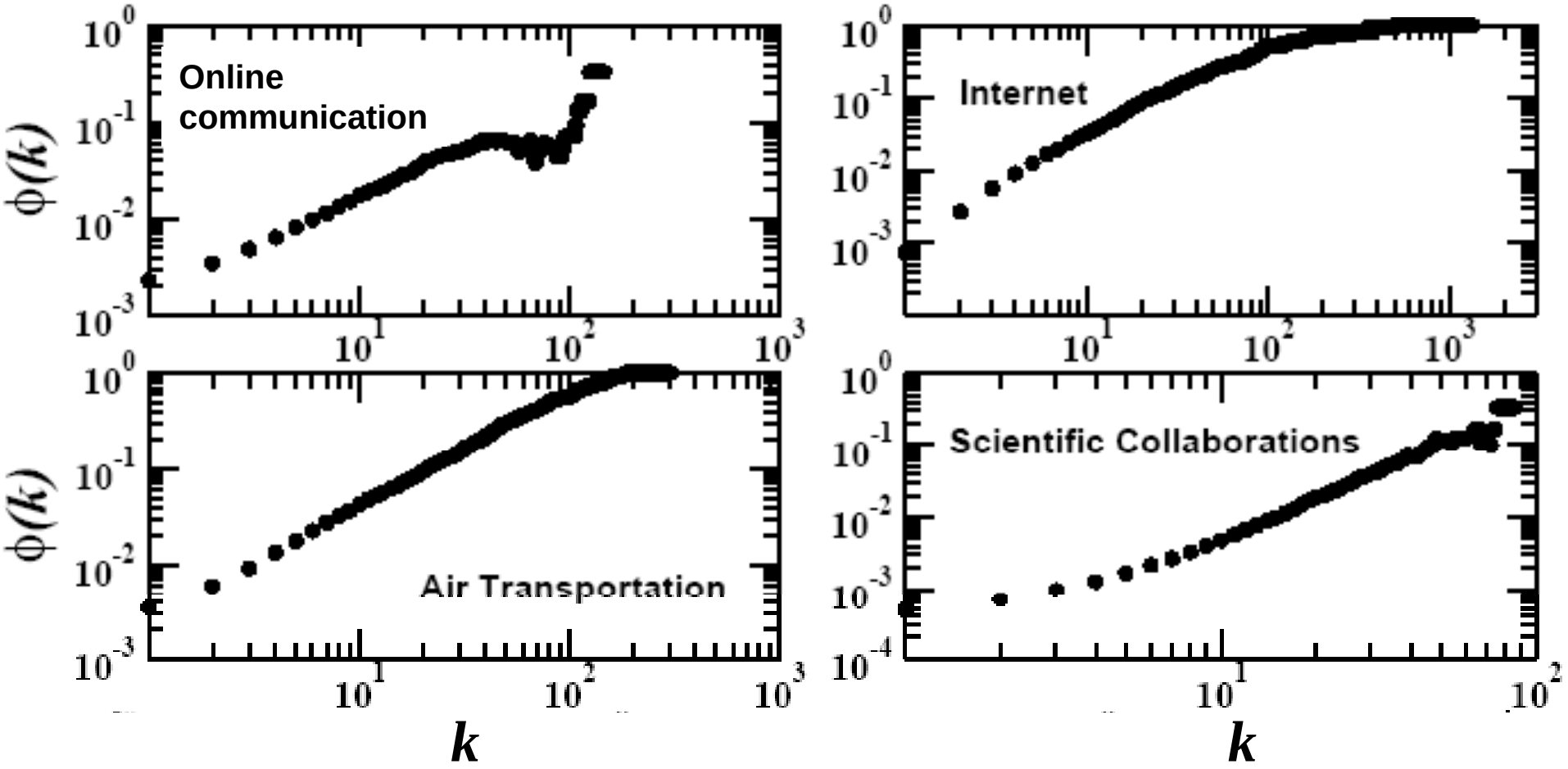


Internet, AS level
(source: CAIDA)

- 11,174 nodes
- 23,409 links

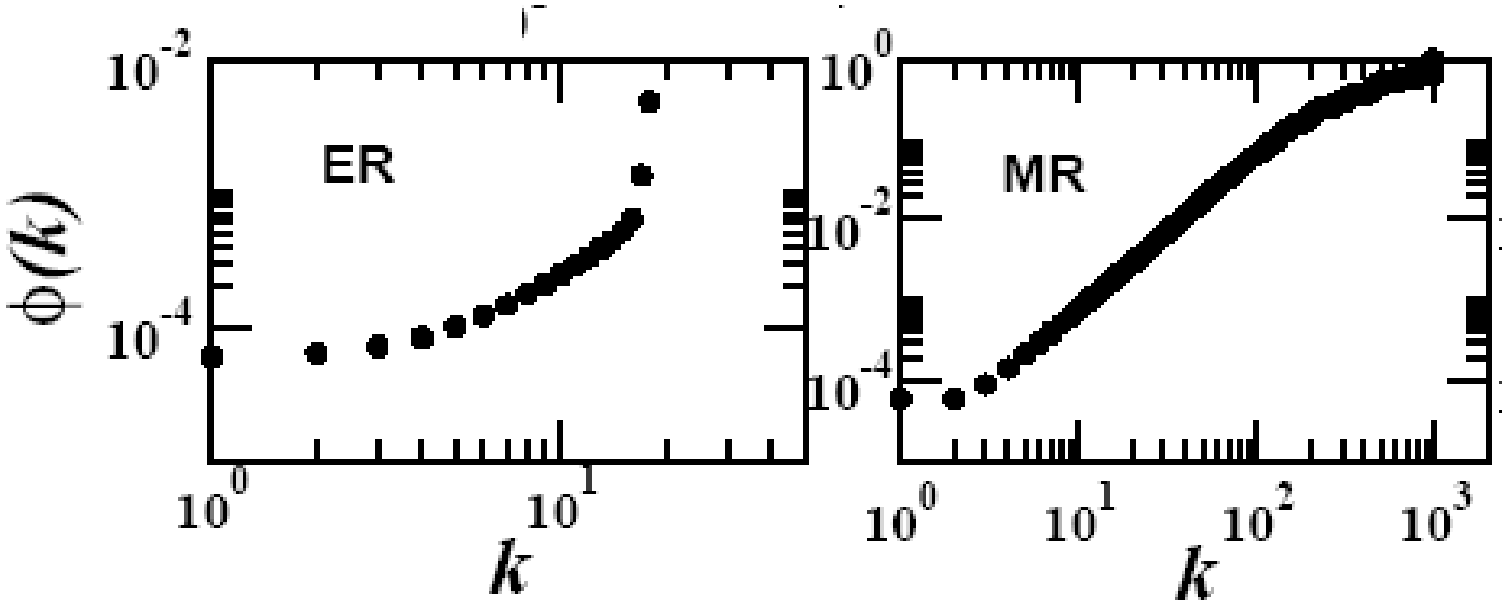
rich-club coefficient

Real-world networks



rich-club coefficient

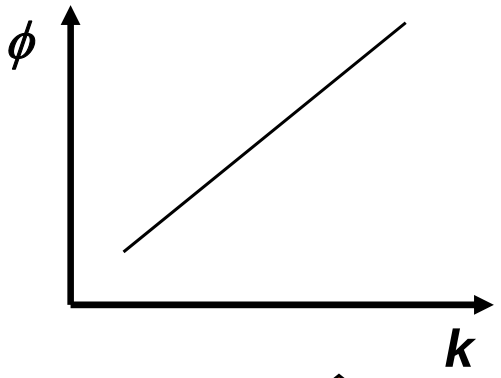
Network models



$\phi(k)$ increases !!! → rich club ???

$\phi(k)$: continuum limit

$$\phi(k) = \frac{N\langle k\rangle \int_k^{k_{\max}} dk' \int_k^{k_{\max}} dk'' P(k',k'')}{\left[N\int_k^{k_{\max}} dk' P(k')\right]\cdot\left[N\int_k^{k_{\max}} dk' P(k') - 1\right]}$$

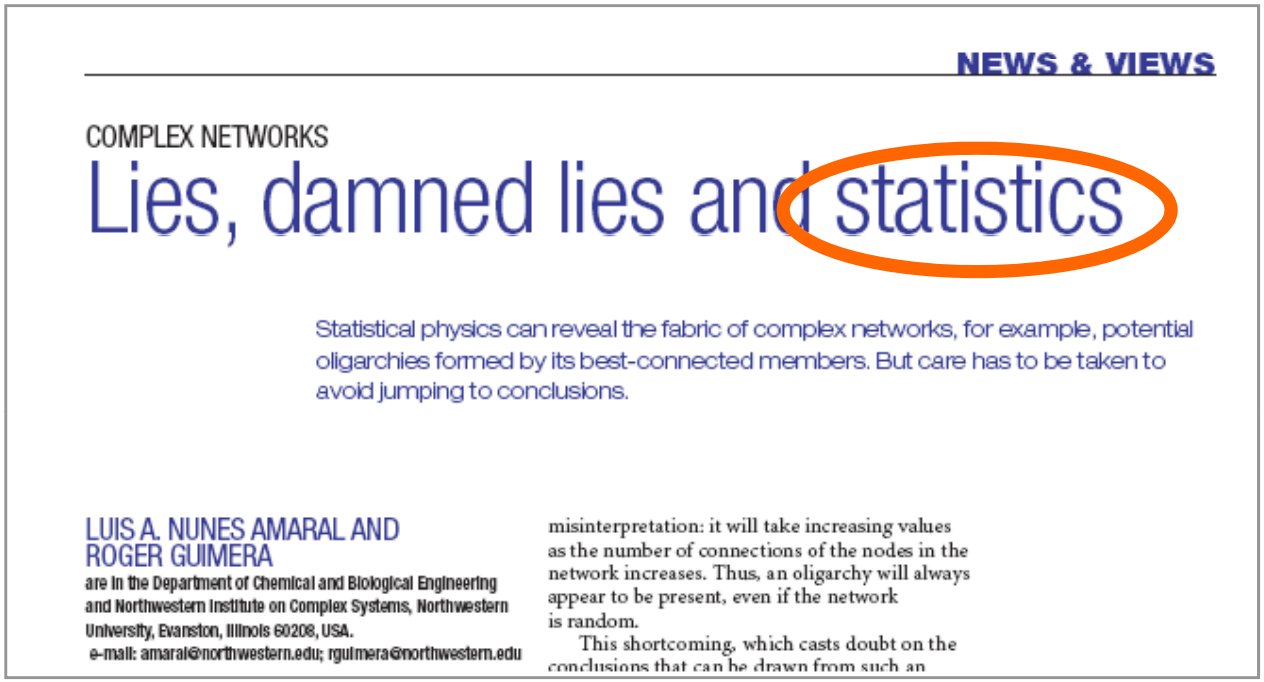


uncorrelated network: $P(k,k') = \frac{kP(k)k'P(k')}{\langle k\rangle^2}$

$$\phi_{unc}(k) = \frac{1}{N\langle k\rangle}\left[\frac{\int_k^{k_{\max}} dk' k' P(k')}{\int_k^{k_{\max}} dk' P(k')}\right]^2 \underset{N\to\infty}{\approx} k^2$$

...lies???

need for a null model



NEWS & VIEWS

COMPLEX NETWORKS

Lies, damned lies and statistics

Statistical physics can reveal the fabric of complex networks, for example, potential oligarchies formed by its best-connected members. But care has to be taken to avoid jumping to conclusions.

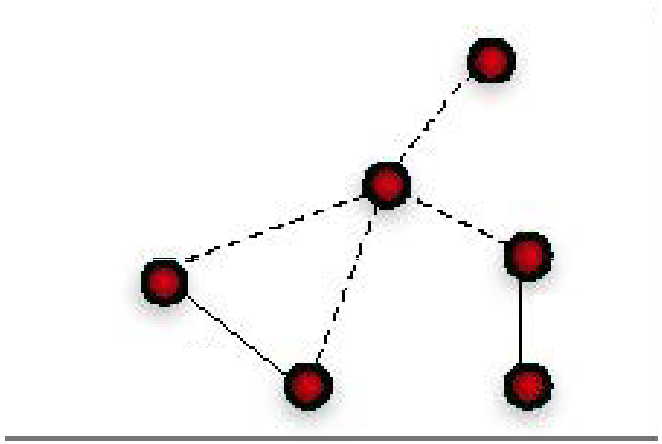
LUIS A. NUNES AMARAL AND ROGER GUIMERA
are in the Department of Chemical and Biological Engineering and Northwestern Institute on Complex Systems, Northwestern University, Evanston, Illinois 60208, USA.
e-mail: amaral@northwestern.edu; rguimera@northwestern.edu

misinterpretation: it will take increasing values as the number of connections of the nodes in the network increases. Thus, an oligarchy will always appear to be present, even if the network is random.

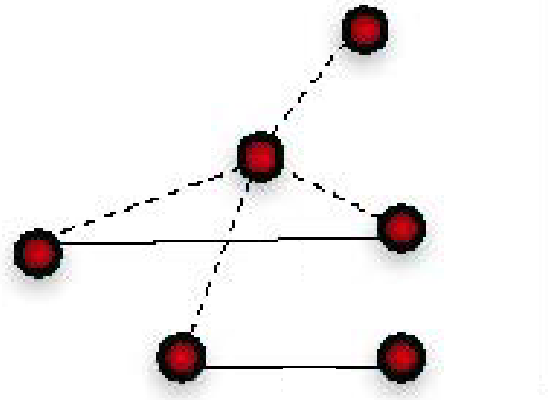
This shortcoming, which casts doubt on the conclusions that can be drawn from such an

null model & normalized ϕ

before



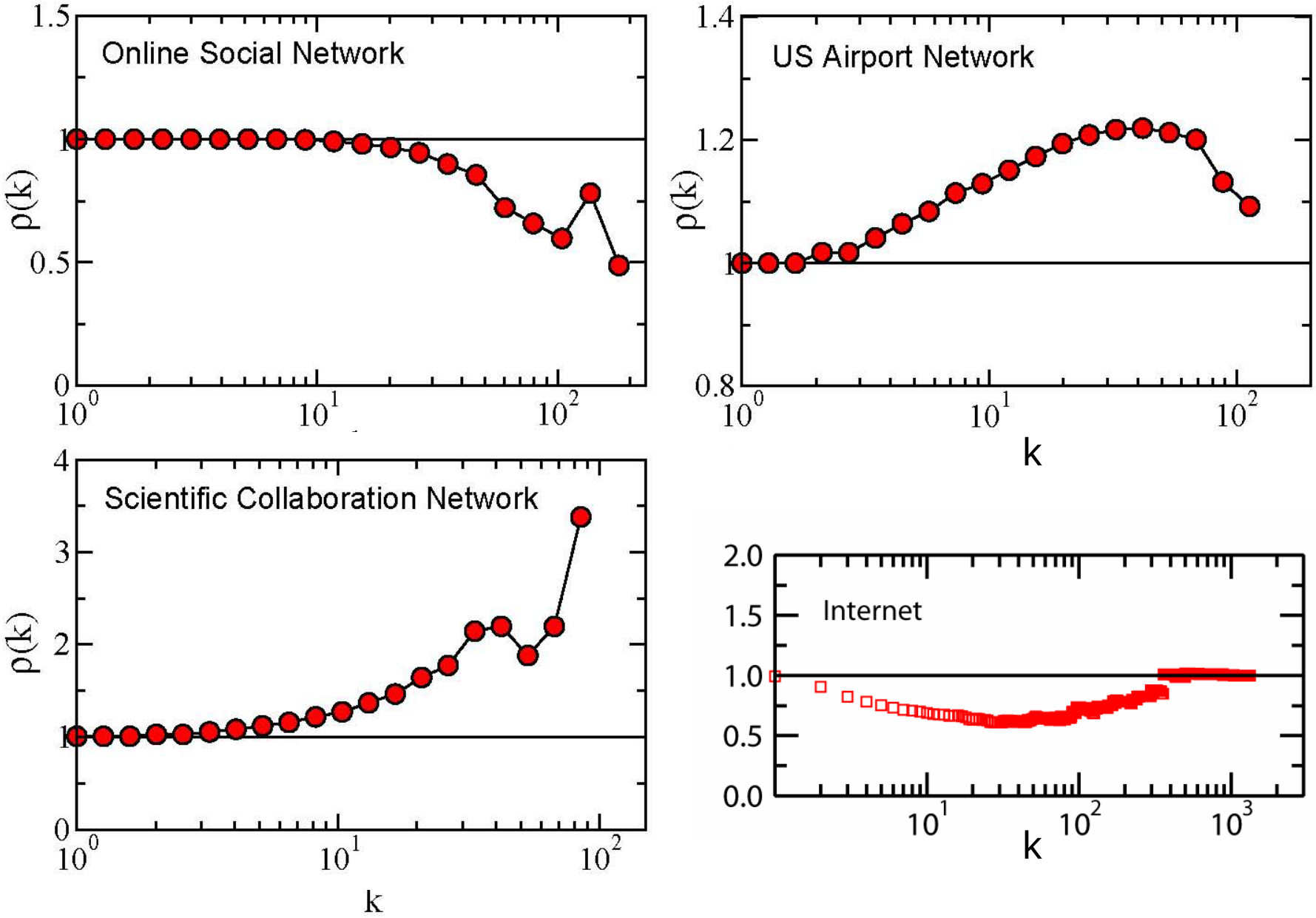
after



$$\Rightarrow \quad \phi_{null}(k), \quad \rho_{null}(k) = \phi(k) / \phi_{null}(k) \begin{cases} > 1 & \textit{rich-club} \\ < 1 & \textit{not rich-club} \end{cases}$$

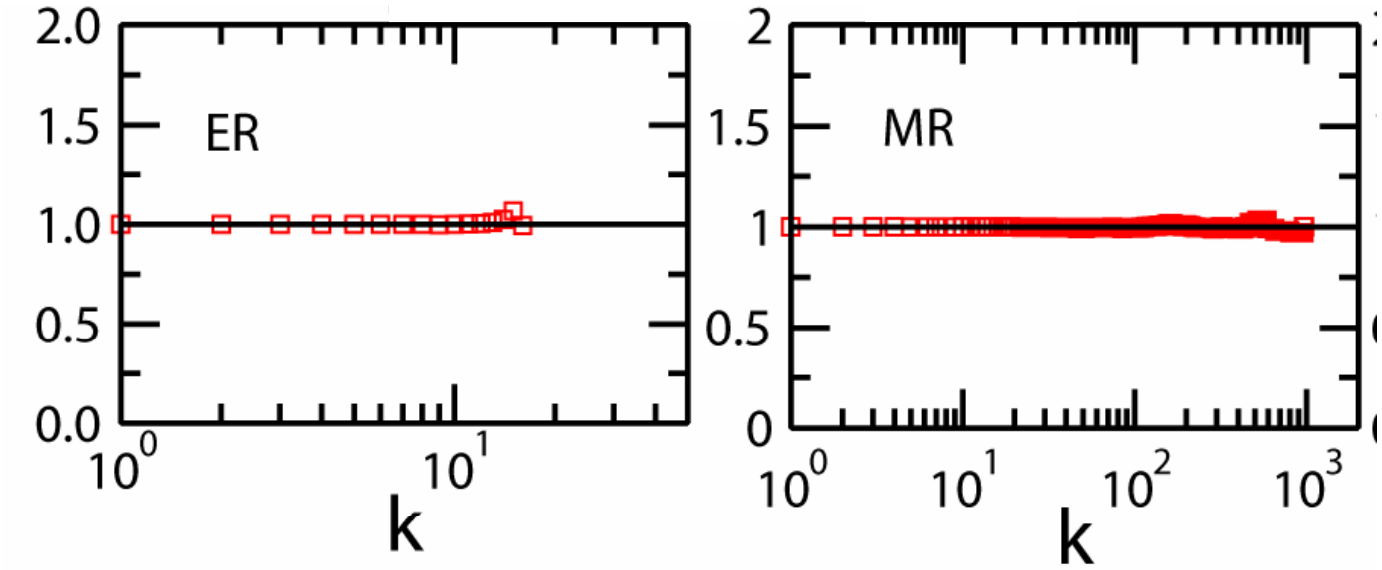
Colizza, Flammini, Serrano, Vespignani. *Nature Phys* (2006)

rich-club phenomenon



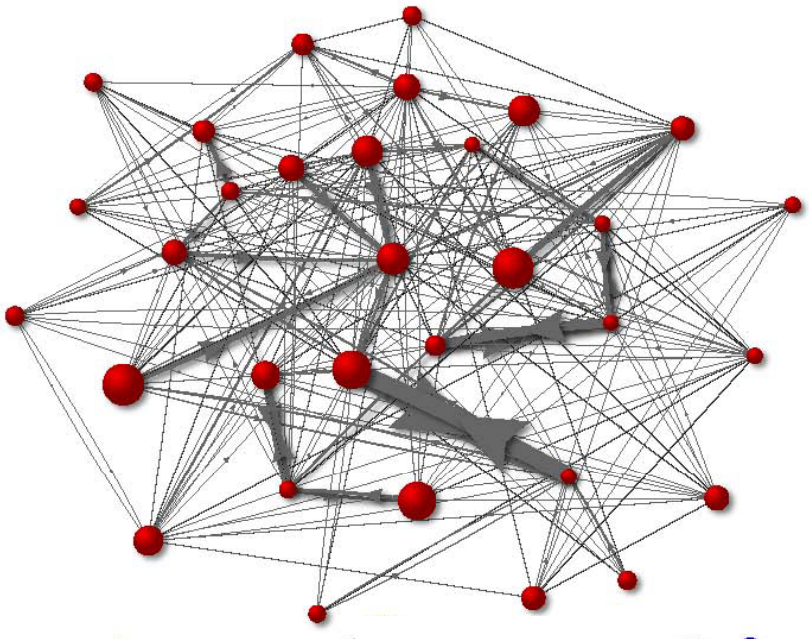
Colizza, Flammini, Serrano, Vespignani. *Nature Phys* (2006)

rich-club phenomenon: just a check...



Colizza, Flammini, Serrano, Vespignani. *Nature Phys* (2006)

...weighted networks?



weight = capacity, intensity of interaction

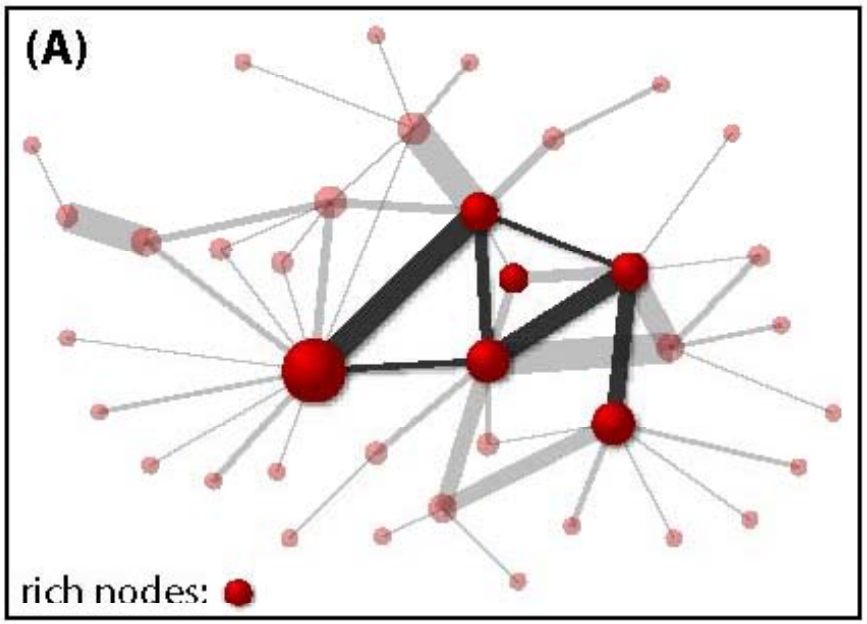
- mobility: # travelers
- collaboration: normalized # papers co-authored
- communication: # messages sent

strength:

$$s_i = \sum_j w_{ij}$$

Barrat et al. *PNAS* (2004)

rich-club → richness parameter *r*

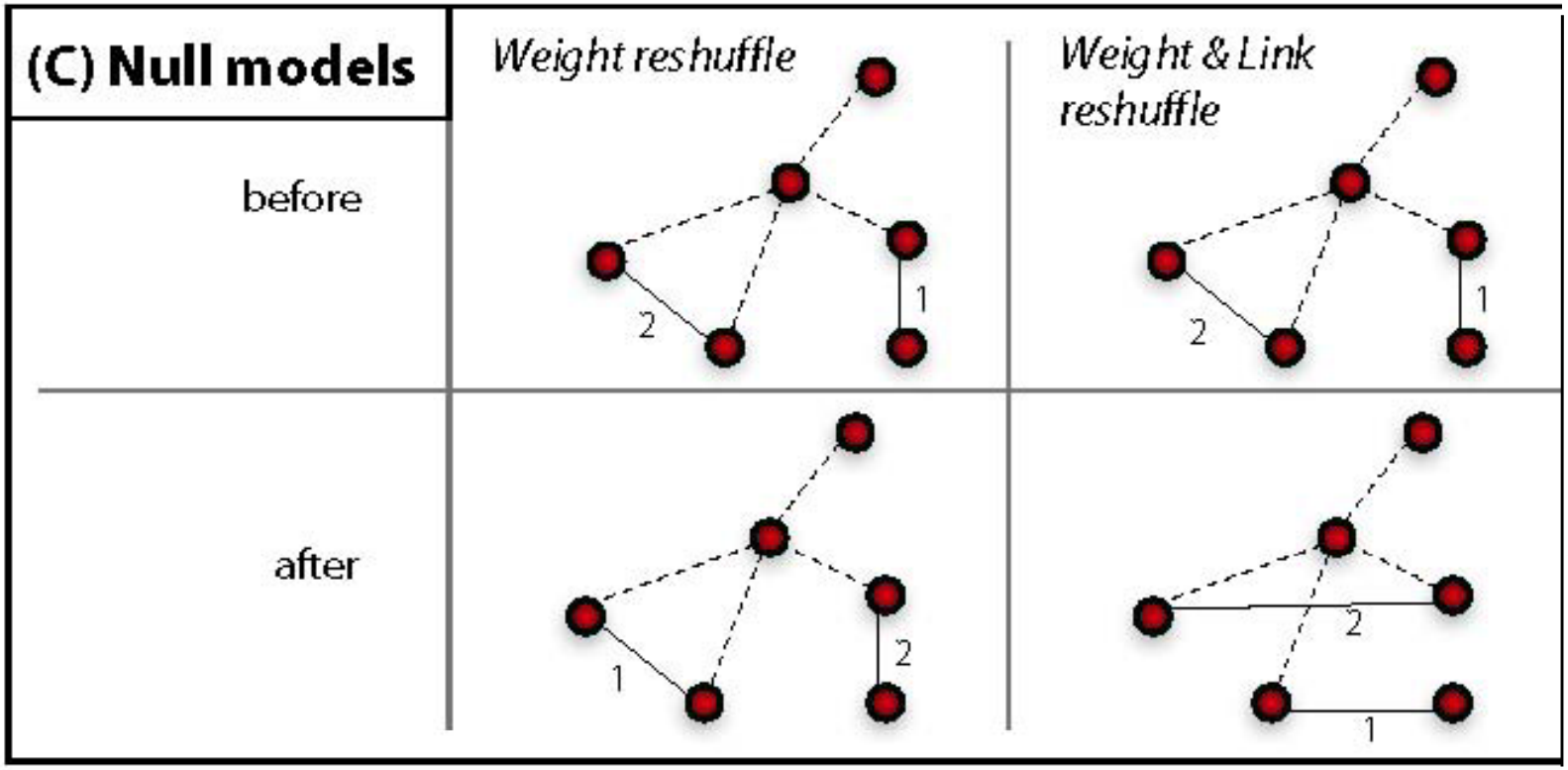


$$\phi^{w}(r) = \frac{W_{>r}}{?}$$

$$w_1^{rank} \geq w_2^{rank} \geq \ldots$$

Opsahl, Colizza, Panzarasa, Ramasco. *PRL* (2008)

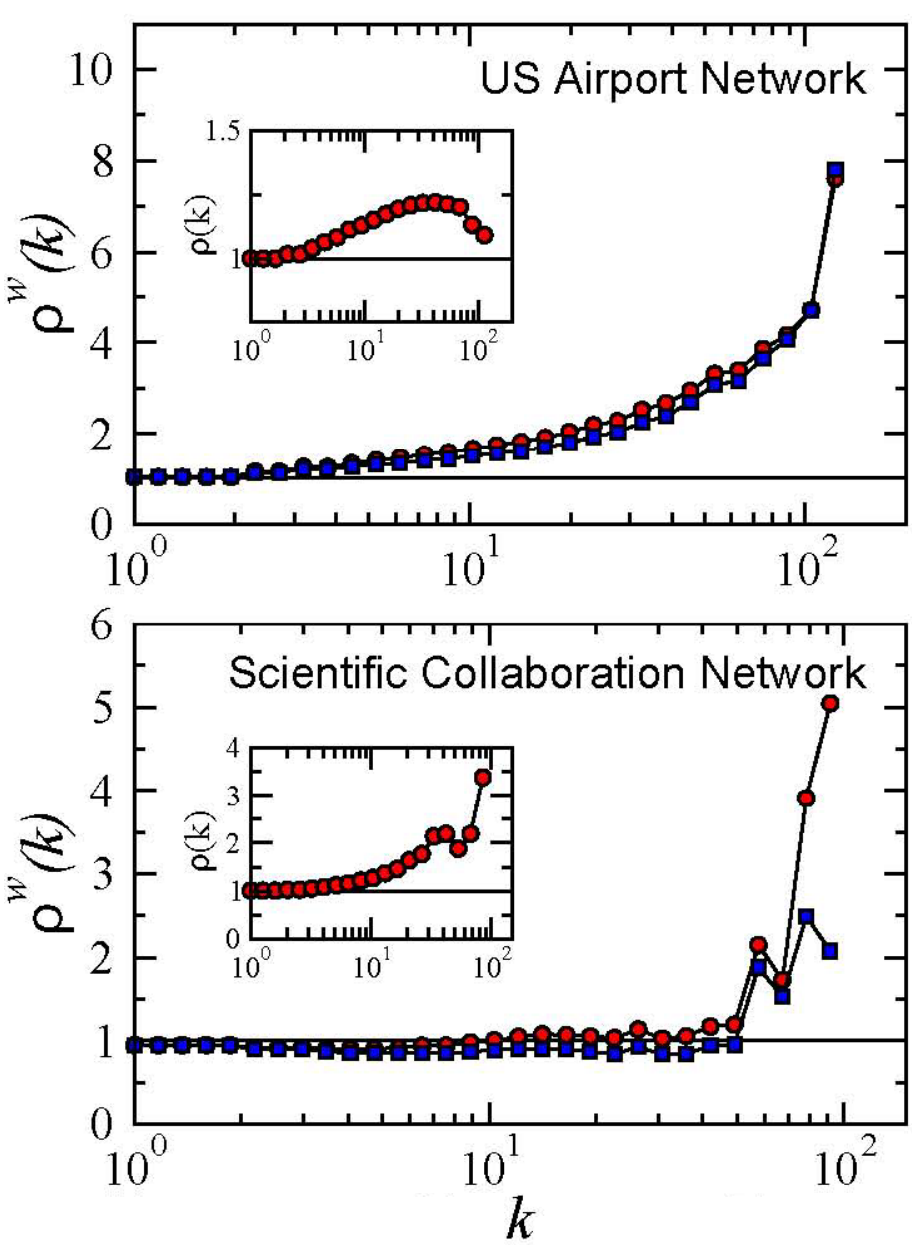
null models

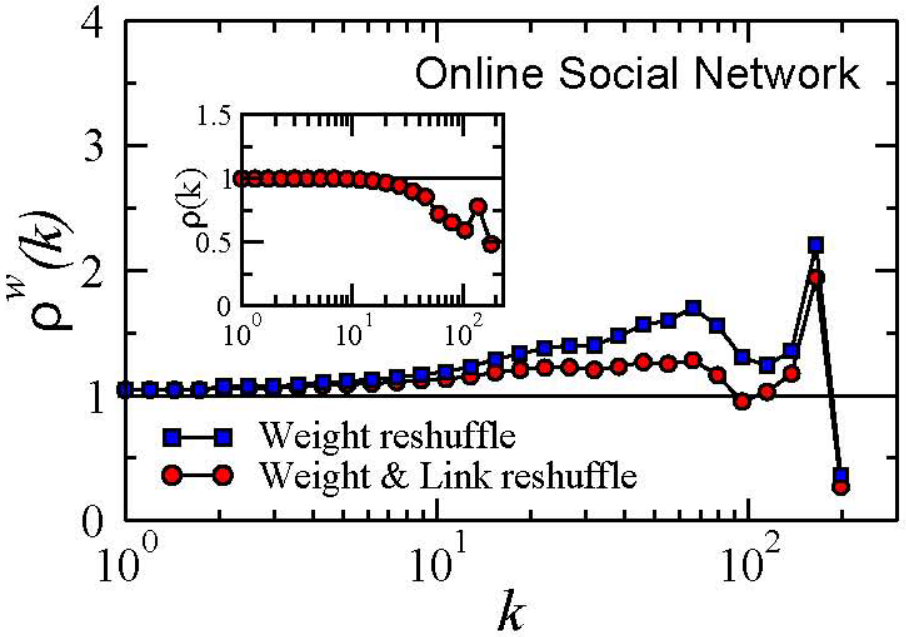


$$\rho^w(r) = \frac{\phi^w(r)}{\phi^w_{null}(r)}$$

Opsahl, Colizza, Panzarasa, Ramasco. *PRL* (2008)

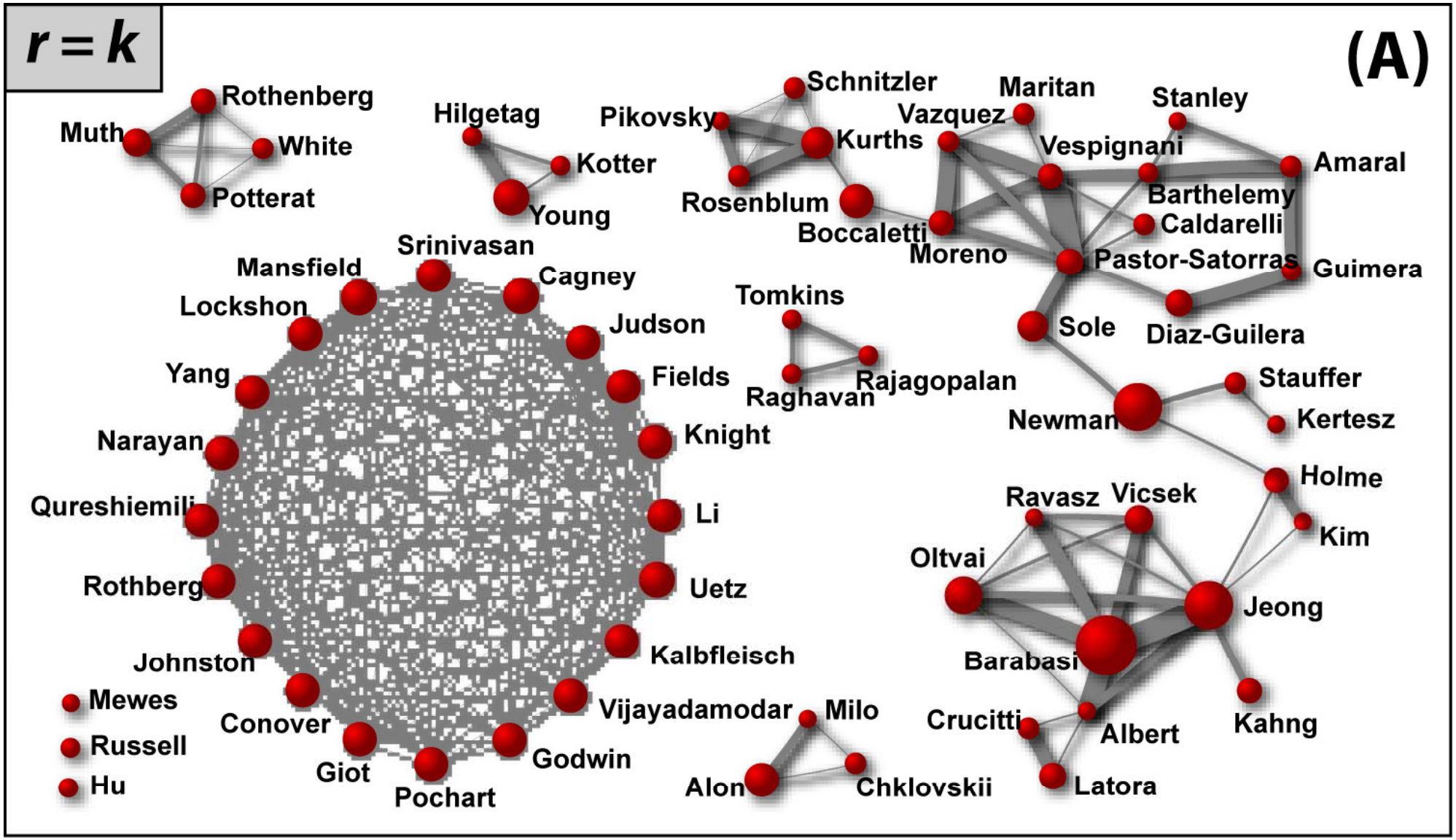
r = k



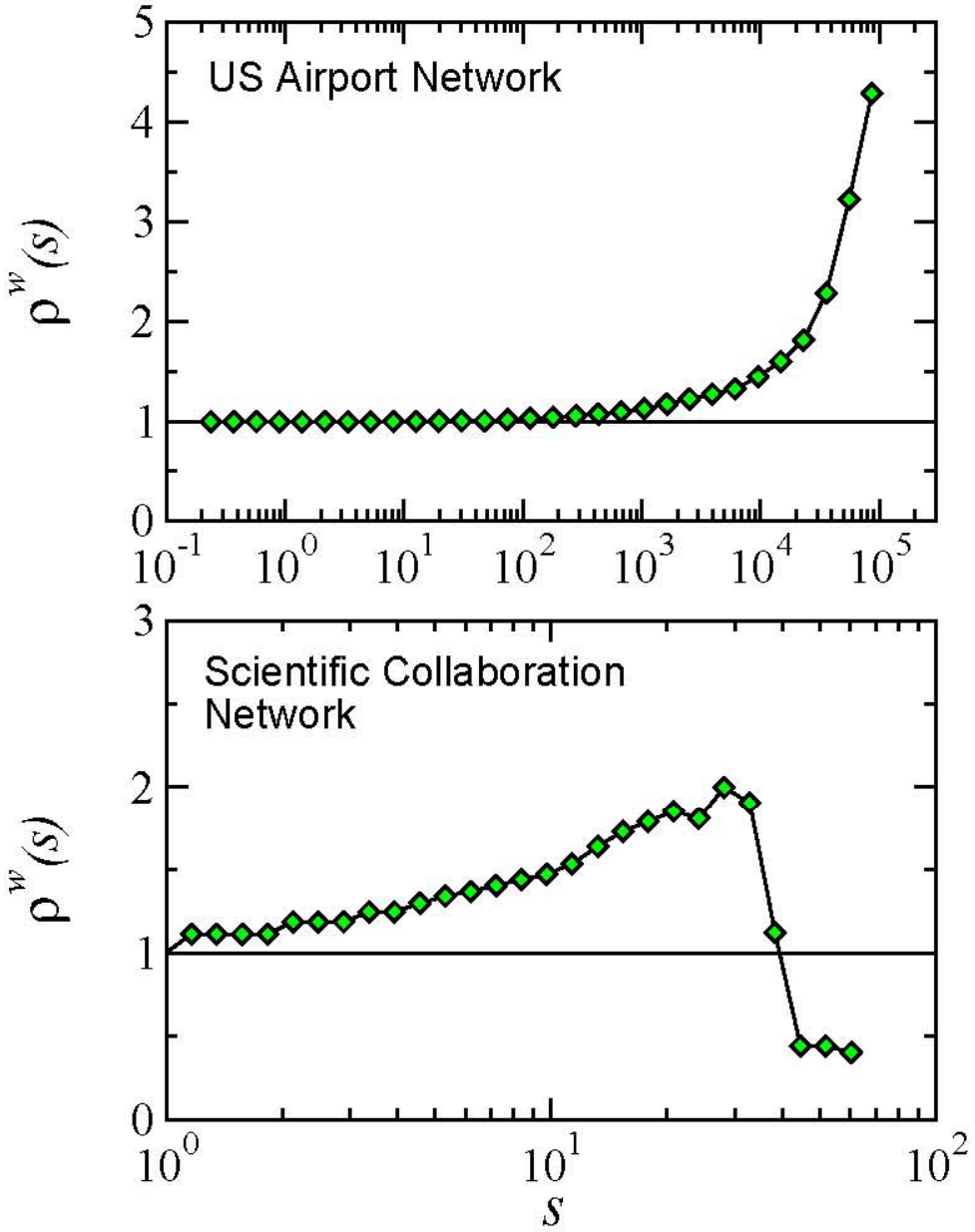


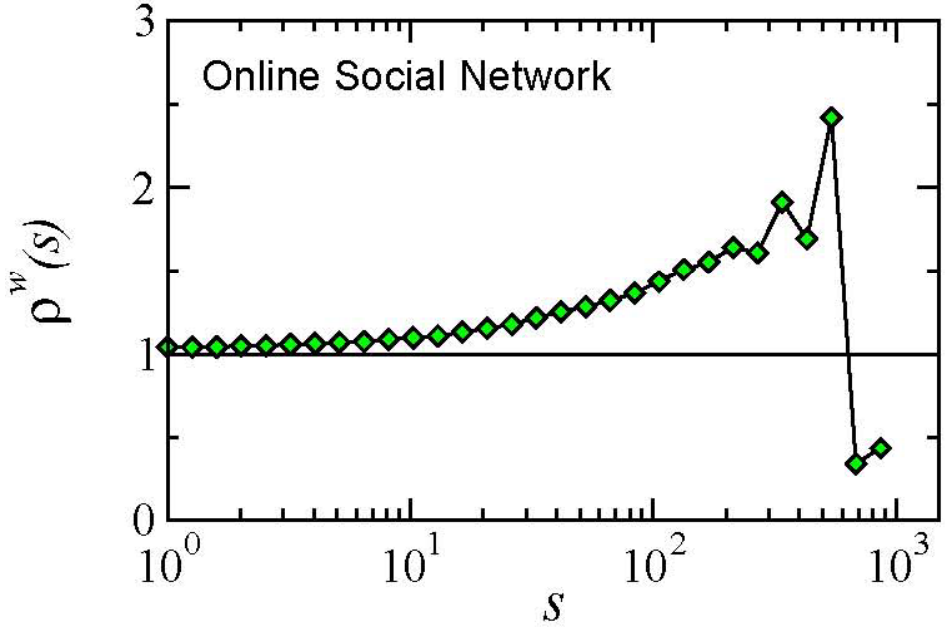
Opsahl, Colizza, Panzarasa, Ramasco. *PRL* (2008)

$r = k$


r = k
(A)
Rothenberg
Muth
White
Potterat
Hilgetag
Kotter
Young
Schnitzler
Pikovsky
Kurths
Rosenblum
Boccaletti
Maritan
Vazquez
Vespignani
Moreno
Stanley
Amaral
Barthelemy
Caldarelli
Pastor-Satorras
Guimera
Sole
Diaz-Guilera
Srinivasan
Mansfield
Cagney
Lockshon
Judson
Yang
Fields
Narayan
Knight
Qureshiemili
Li
Rothberg
Uetz
Johnston
Kalbfleisch
Mewes
Russell
Hu
Conover
Vijayadamodar
Giot
Godwin
Pochart
Tomkins
Raghavan
Rajagopalan
Stauffer
Newman
Kertesz
Holme
Ravasz
Vicsek
Kim
Oltvai
Jeong
Barabasi
Crucitti
Albert
Kahng
Latora
Milo
Alon
Chklovskii

r = s



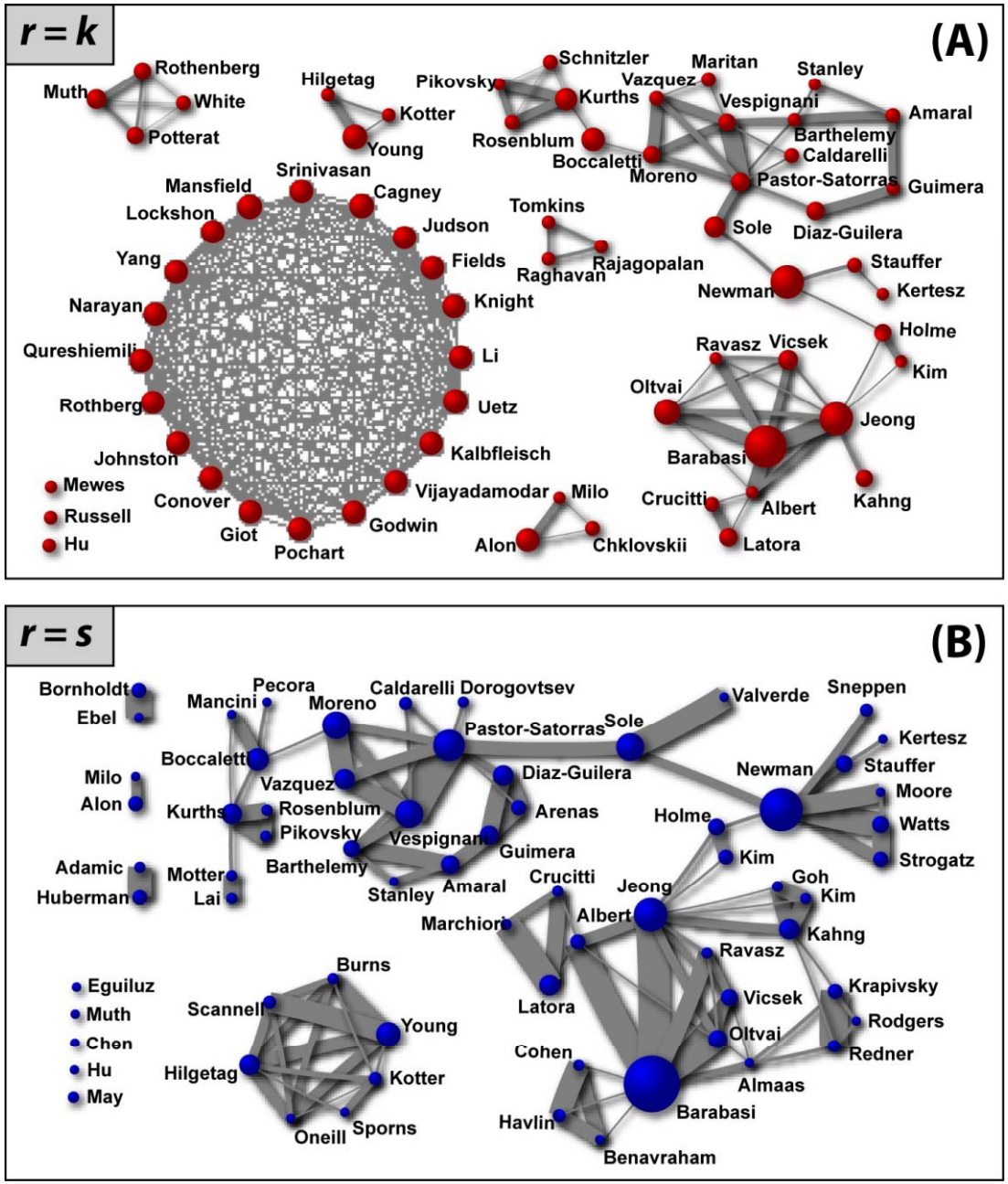


strength:

$$s_i = \sum_j w_{ij}$$

Opsahl, Colizza, Panzarasa, Ramasco. *PRL* (2008)

$r = s$


r = k
(A)
Rothenberg
Muth
White
Potterat
Hilgetag
Kotter
Young
Pikovsky
Schnitzler
Kurths
Rosenblum
Boccaletti
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Newman
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Jeong
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Cruciti
Albert
Kahng
Latora
r = s
(B)
Bornholdt
Ebel
Pecora
Mancini
Moreno
Caldarelli
Dorogovtsev
Pastor-Satorras
Sole
Valverde
Sneppen
Kertesz
Stauffer
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Watts
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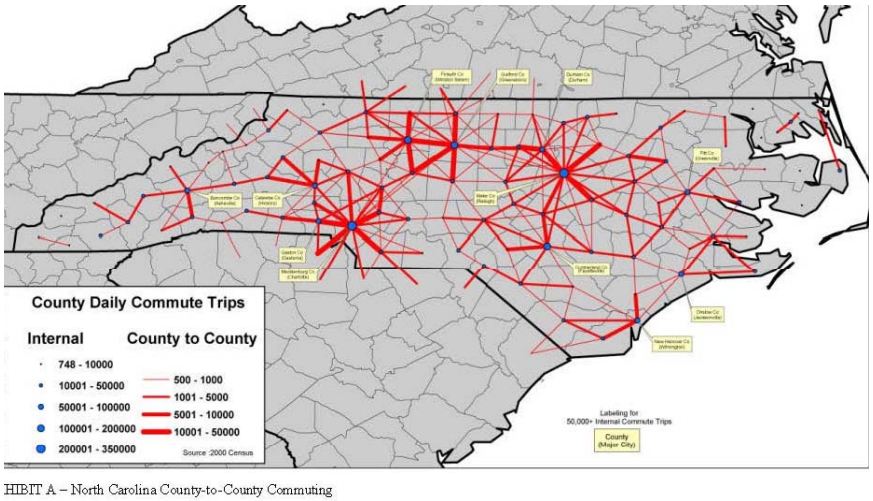
Ravasz
Latora
Eguiluz
Muth
Chen
Hu
May
Burns
Scannell
Young
Hilgetag
Kotter
Oneill
Sporns
Krapivsky
Vicsek
Rodgers
Oltvai
Redner
Cohen
Almaas
Barabasi
Havlin
Benavraham

focus on human mobility

US airport network
676 airports

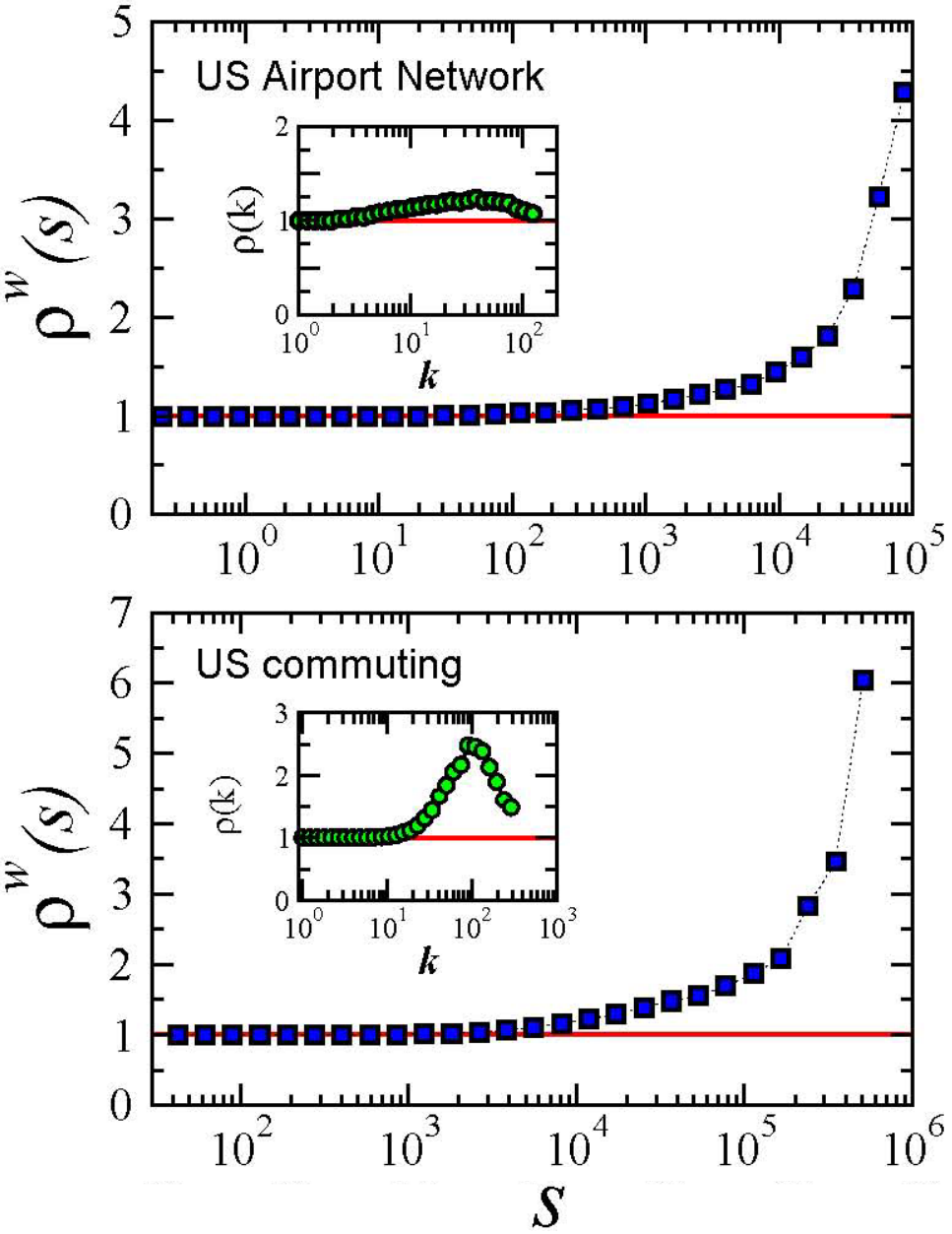


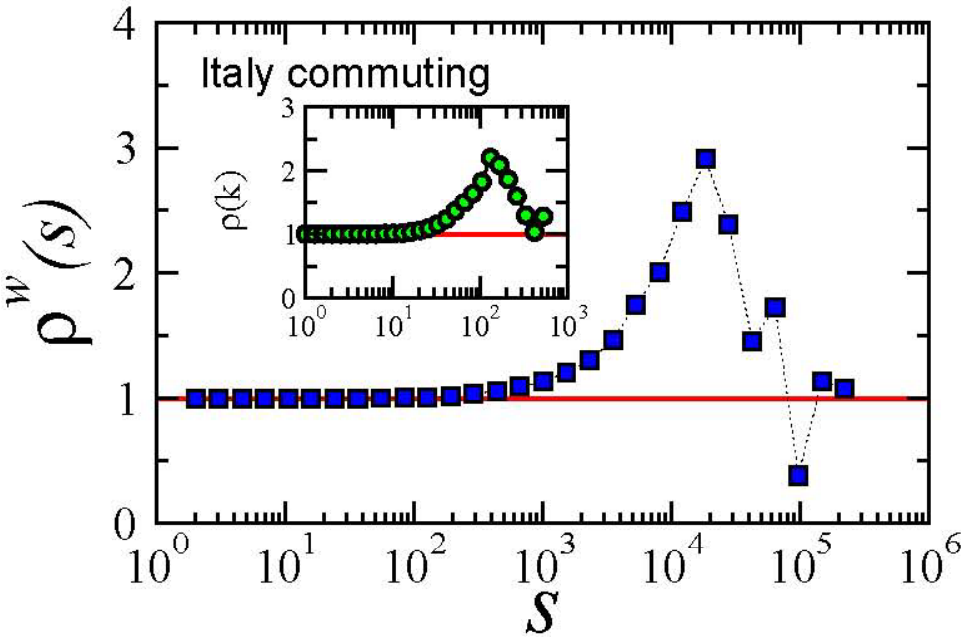
HIBIT A – North Carolina County-to-County Commuting

US census commuting flows
3141 counties
link: residence - workplace
weight: # commuters

IT municip. commuting flows
8101 municipalities

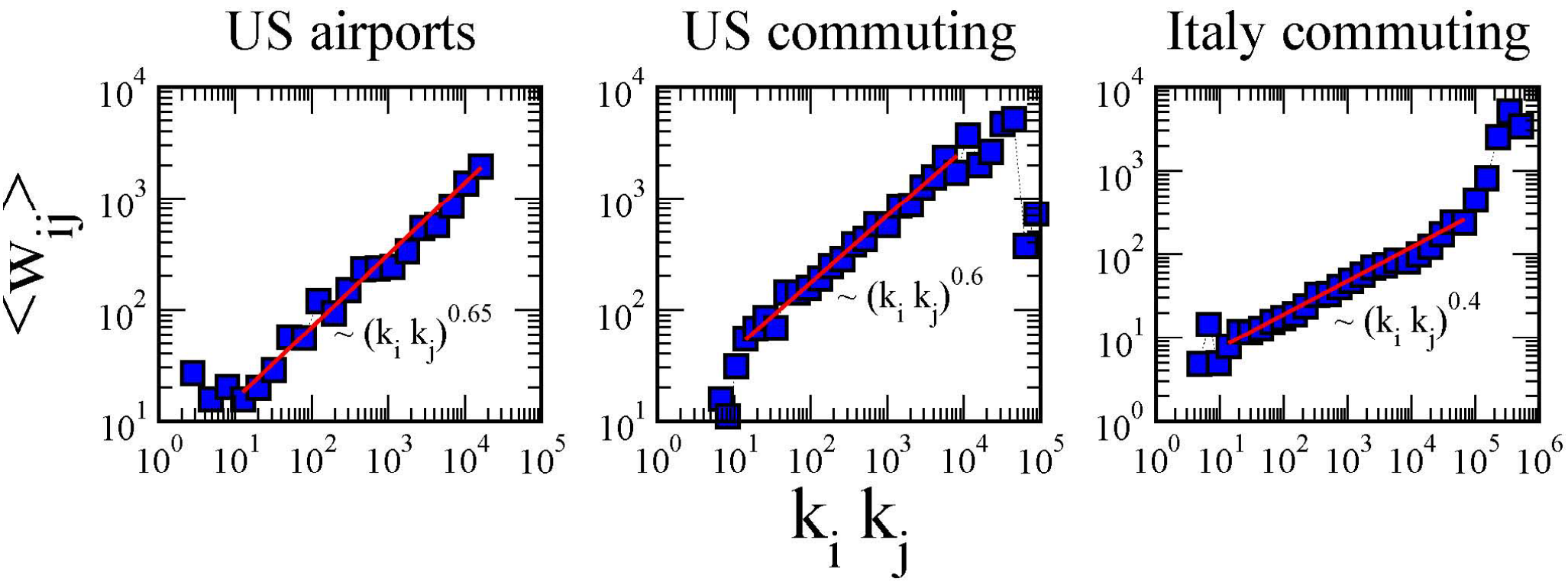
r = s





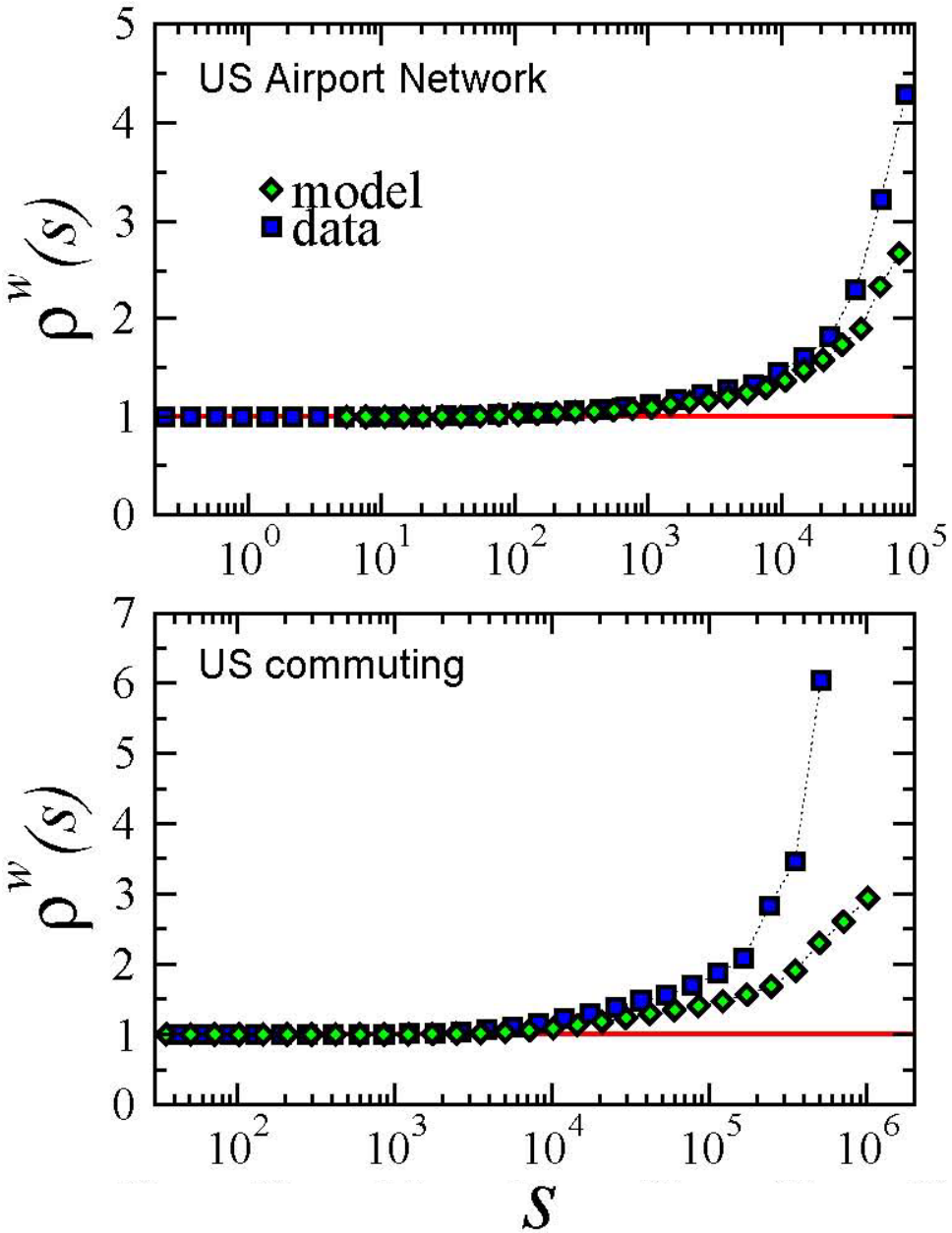
Ramasco, Colizza, Panzarasa. *work in progress.*

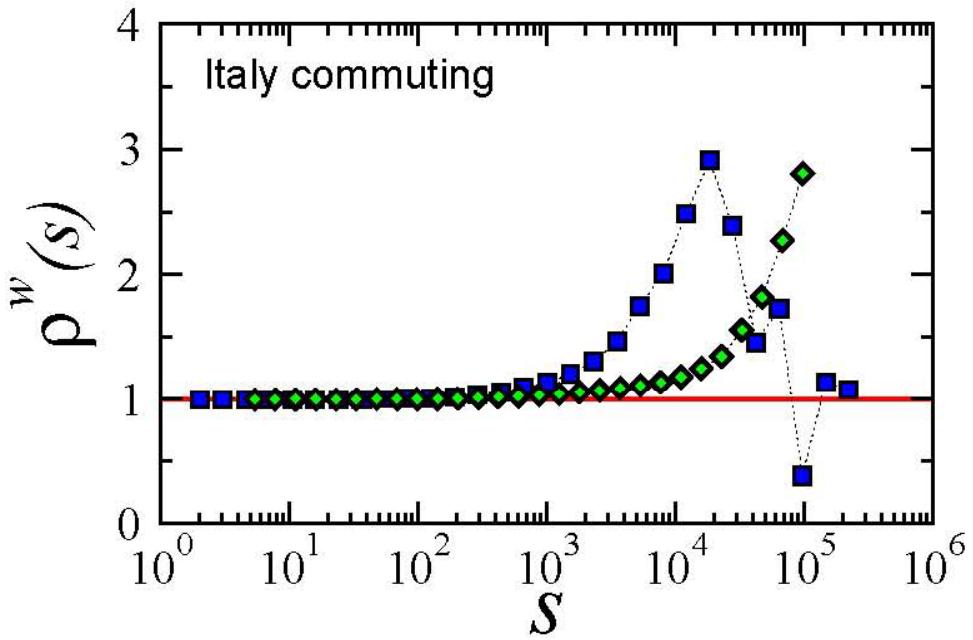
a simple mobility model



Balcan, Colizza, Goncalves, Hu, Ramasco, Vespignani, *work in progress.*
Ramasco, Colizza, Panzarasa. *work in progress.*

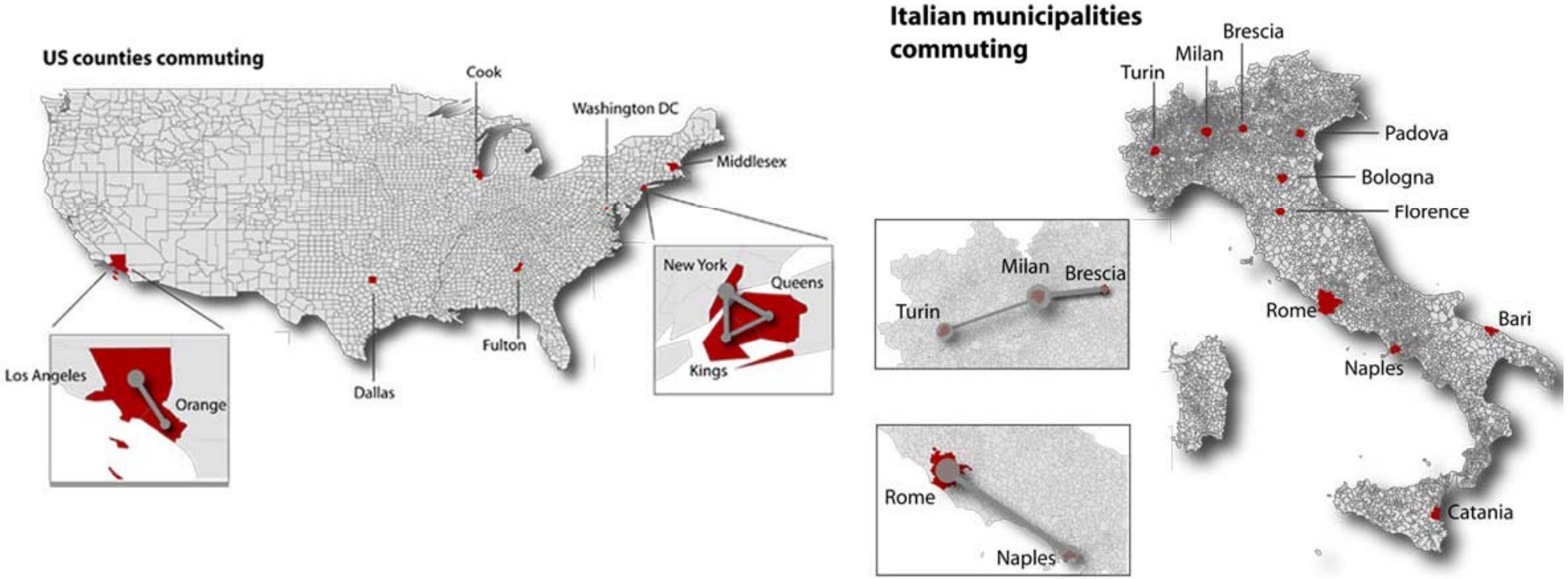
r = s





Ramasco, Colizza, Panzarasa. *work in progress.*

differences in mobility



Balcan, Colizza, Goncalves, Hu, Ramasco, Vespignani, *work in progress.*
Ramasco, Colizza, Panzarasa. *work in progress.*

acknowledgments & refs.

Mariangeles Serrano *(IFISC, Spain)*
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Jose Ramasco *(ISI Foundation)*
Tore Opsahl *(Queen Mary Univ., London)*
Pietro Panzarasa *(Queen Mary Univ.)*

cxnets.googlepages.com

Colizza, Flammini, Serrano, Vespignani. *Nature Phys* (2006)
Opsahl, Colizza, Ramasco, Panzarasa. *Phys Rev Lett* (2008)

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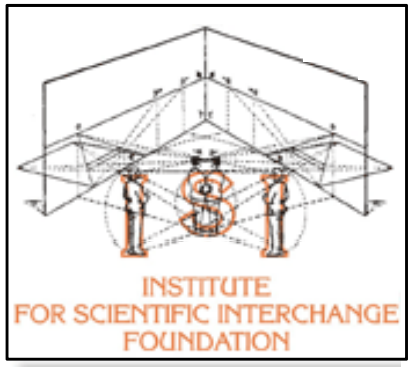

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